LINES AND ANGLES

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Introduction

- Line is a straight one-dimensional figure having no thickness and extending infinitely in both directions. E.g. Fig. 1.1 shows line AB.

Fig. 1.1

- Angle is a shape formed by two lines or rays diverging from a common point (the vertex). E.g. Fig. 1.2 shows angle AOB.

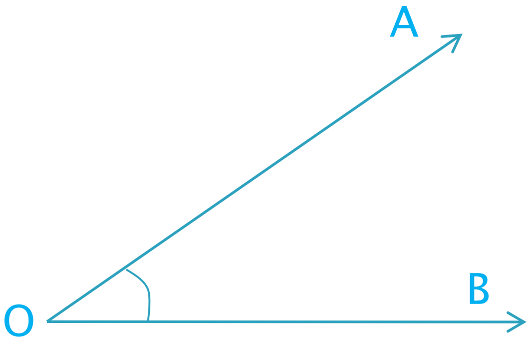


Fig. 1.2

Introduction (Contd..)

- In this chapter, you will study the properties of the angles formed when two lines intersect each other and also the properties of the angles formed when a line intersects two or more parallel lines at distinct points.
- You will also study about line parallel to the same line and angle sum property of a triangle.

Basic Terms and Definitions

- **Line Segment** – A part of a line with two end points is called a line segment. E.g. Fig. 1.3 shows line segment AB.

Fig. 1.3

- **Ray** – A part of a line with one end point is called a ray. E.g. Fig. 1.4 shows ray AB.

Fig. 1.4

Basic Terms and Definitions (Contd..)

- **Collinear and Non-Collinear Points** – If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear points. E.g. In Fig. 1.5, A, B, C are collinear points and D, E, F are non-collinear points.

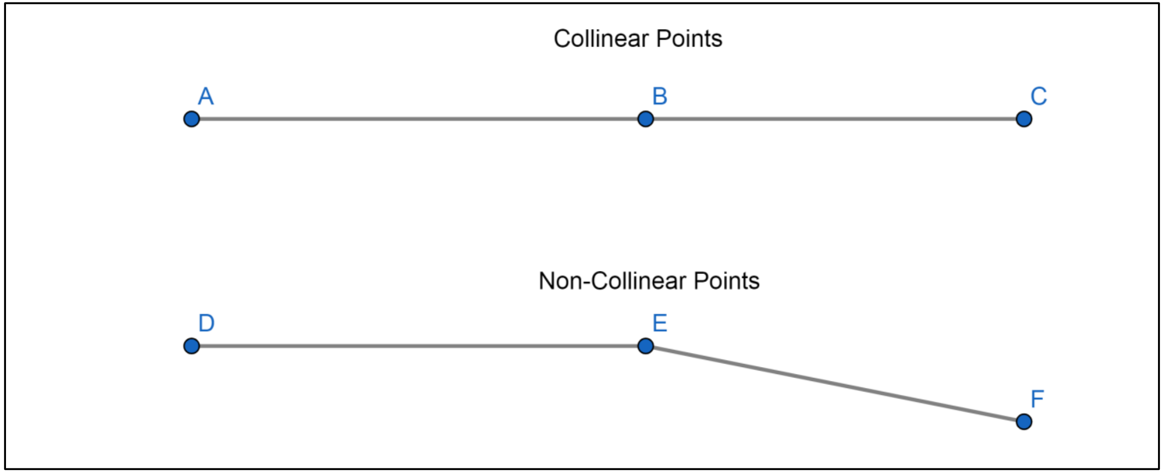
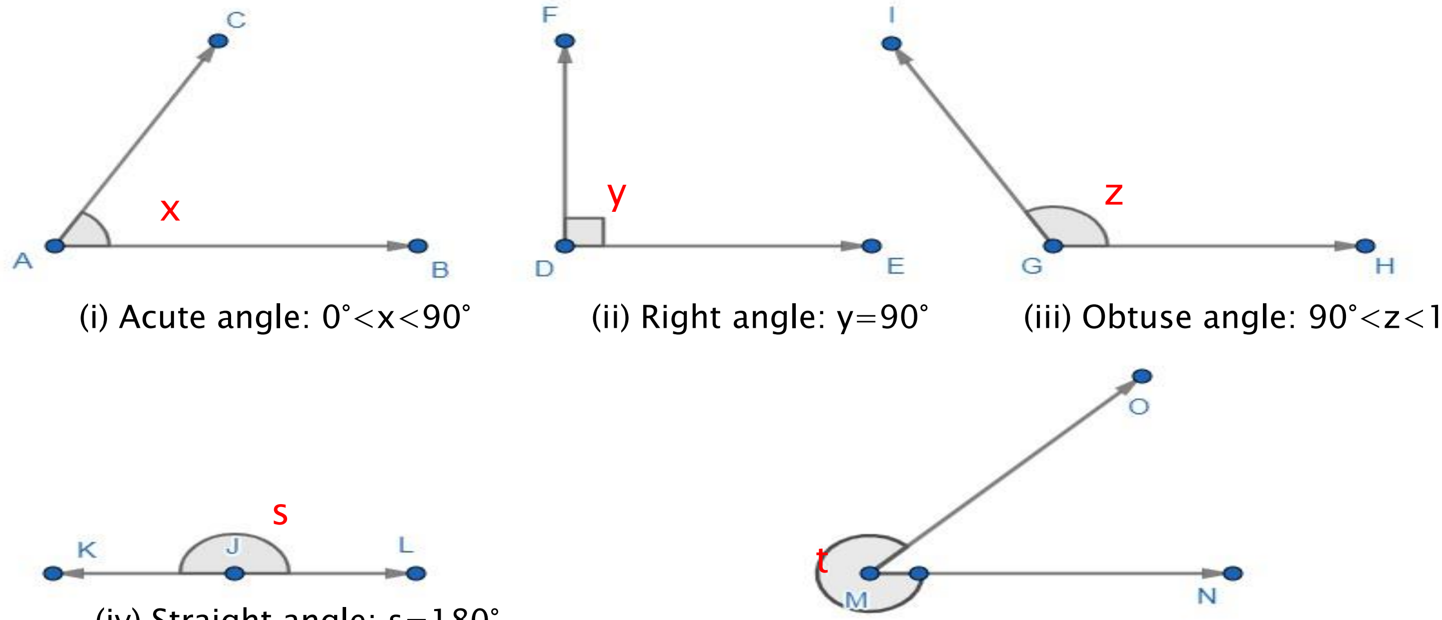


Fig. 1.5

Basic Terms and Definitions (Contd..)

- **Types of Angles** – Different types of angles are given in Fig. 1.6:

Fig. 1.6



(i) Acute angle: 0°<x<90°

(ii) Right angle: y=90°

(iii) Obtuse angle: 90°<z<180°

(iv) Straight angle: s=180°

(v) Reflex angle: 180 ° <t<360°

Basic Terms and Definitions (Contd..)

- **Complementary Angles** – Two angles whose sum is 90° are called complementary angles.
- **Supplementary Angles** – Two angles whose sum is 180° are called supplementary angles.
- **Adjacent Angles** – Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of the common arm. In Fig. 1.7, ∠ABD and ∠DBC are adjacent angles.

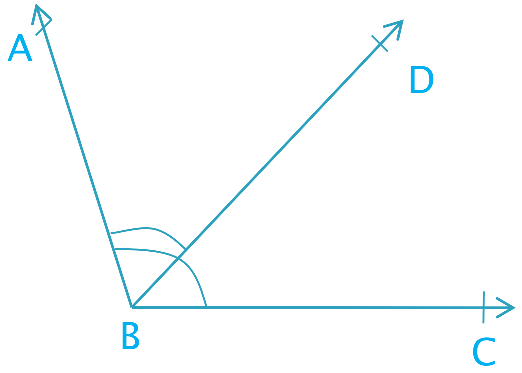


Fig. 1.7

Basic Terms and Definitions (Contd..)

- **Linear pair of Angles** – If non-common arms of adjacent angles form a line, then those angles are called linear pair of angles. E.g. In Fig. 1.8, ∠ABD and ∠DBC are called linear pair of angles
- **Vertically opposite angles** – When two lines intersect each other at a point, two pairs of vertically opposite angles are formed. E.g. In Fig. 1.9, AB and CD intersect each other at O to form two pairs of vertically opposite angles.

Fig. 1.8

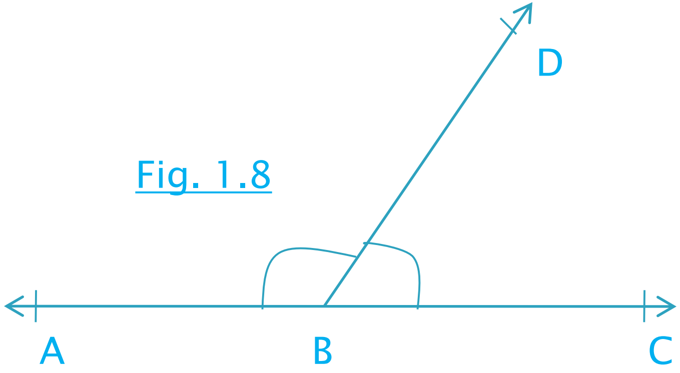
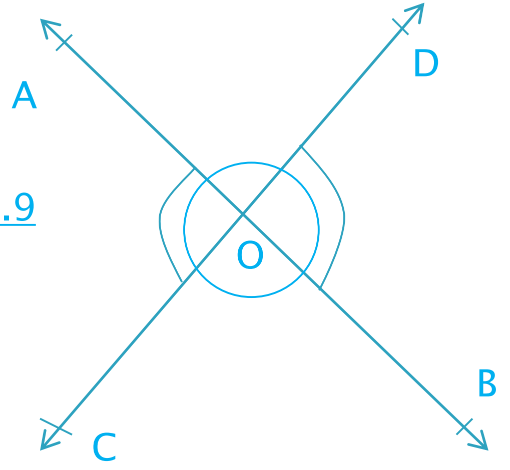


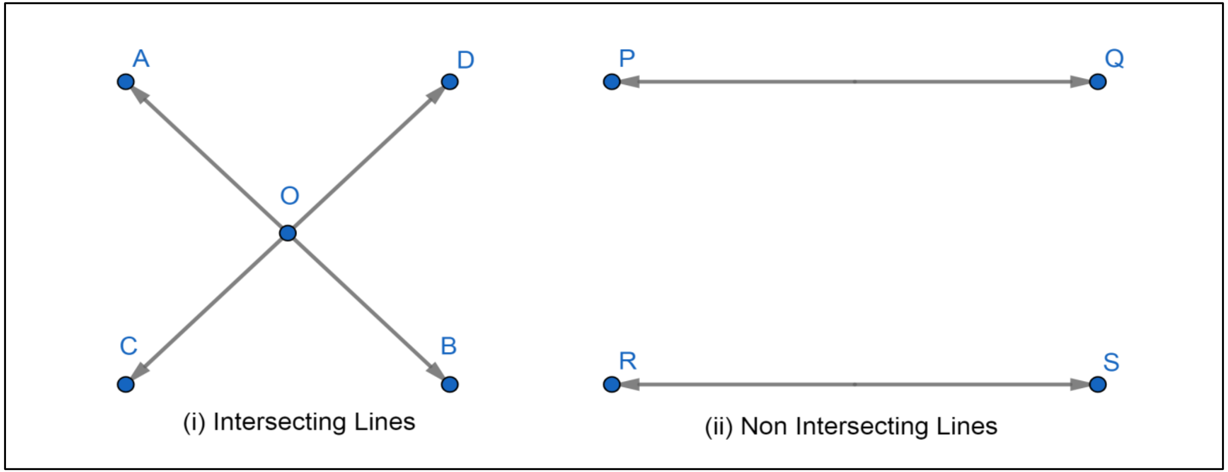
Fig. 1.9



Intersecting Lines and Non Intersecting Lines

- <u>Intersecting Lines</u> – Two lines which intersect each other at a point are called intersecting lines. E.g. In Fig. 1.10, AB and CD are intersecting lines.
- <u>Non Intersecting Lines</u> – Two lines which do not intersect each other at any point and distance between them remains the same are called non intersecting or parallel lines. E.g. In Fig. 1.10, PQ and RS are non-intersecting lines.

Fig. 1.10



Pair of Angles

- We have already discussed the meanings of linear pair of angles and vertically opposite angles. Now we will talk about some axioms and theorems related to different pair of angles.

- Axiom 1: If a ray stands on a line then the sum of two adjacent angles so formed is 180°. E.g. In Fig. 1.11, ray OC stands on line AB, so the sum of two adjacent angles i.e. ∠AOC and ∠COB is 180° by Axiom 1.

- Axiom 2: If the sum of two adjacent angles is 180°, then the two non-common arms of the angles form a line. E.g. In Fig. 1.11, sum of two adjacent angles i.e. ∠AOC and ∠COB is 180°, then the two non-common arms of the angles i.e. AO and OB form a line by Axiom 2.

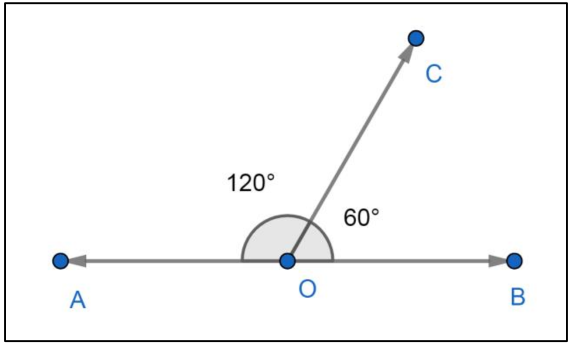


Fig. 1.11

Pair of Angles (Contd..)

- <u>Theorem 1</u>: If two lines intersect each other, then the vertically opposite angles are equal.

  Proof: Let two lines AB and CD intersect each other at O as shown in Fig. 1.12.

Here, two pairs of vertically opposite angles are: (i) $\angle AOD$ and $\angle COB$, (ii) $\angle AOC$ and $\angle DOB$

Now, we can see that, ray OA stands on line CD. Then by Axiom 1 (as discussed earlier), $\angle AOC + \angle AOD = 180°$ (Eq. 1)

Similarly, ray OD stands on line AB. Then, $\angle DOB + \angle AOD = 180°$ (Eq. 2)

By Eq.1 and 2, we can say that
$\angle AOC = \angle DOB = 180° - \angle AOD$
Similarly, we can prove that $\angle AOD = \angle COB$.

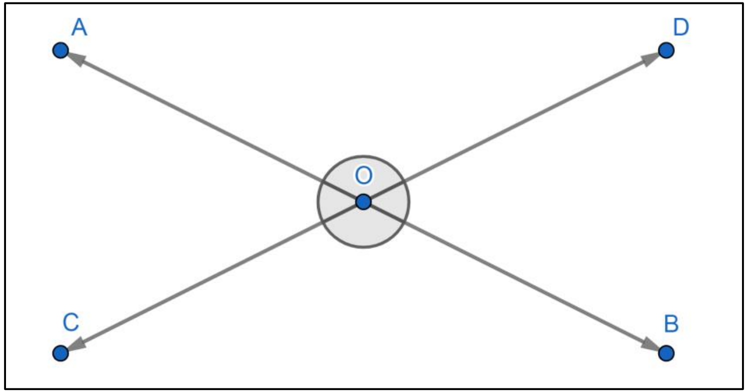


Fig. 1.12

Parallel Lines and a Transversal

- A line which intersects two or more lines at distinct points is called a **transversal.**
- Various pair of angles are formed when a transversal intersects two lines. Let us understand them with an example.

In Fig. 1.13, lines m and n are two parallel lines and transversal l intersect line m at P and line n at Q. Pairs of angles formed are:

a) **Corresponding angles:**
(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$
(iii) $\angle 4$ and $\angle 8$ (iv) $\angle 3$ and $\angle 7$

b) **Alternate interior angles:**
(i) $\angle 4$ and $\angle 6$ (ii) $\angle 3$ and $\angle 5$

c) **Alternate exterior angles:**
(i) $\angle 1$ and $\angle 7$ (ii) $\angle 2$ and $\angle 8$

d) **Interior angles on the same side of the transversal**
(i) $\angle 4$ and $\angle 5$ (ii) $\angle 3$ and $\angle 6$

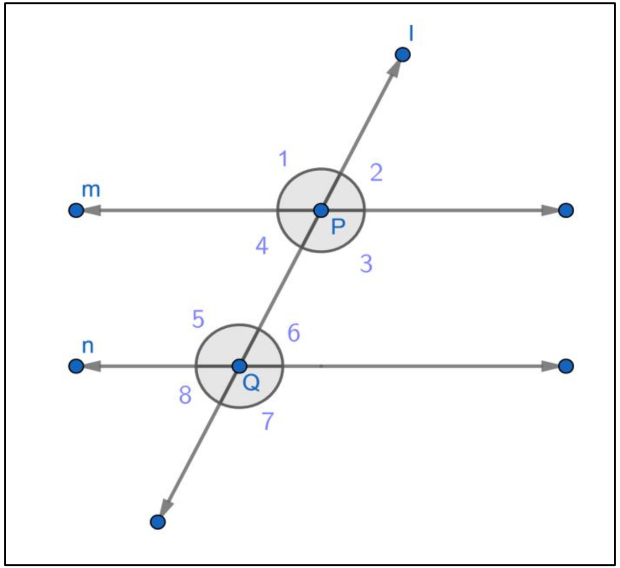


Fig. 1.13

Parallel Lines and a Transversal (Contd..)

- Now we will learn some axioms and theorems related to parallel lines intersected by a transversal.

- <u>Axiom 3</u>: If a transversal intersects two parallel lines, then each pair of corresponding angles is equal. This Axiom is also referred as **corresponding angles axiom**.

- <u>Axiom 4</u>: If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other. This is converse of corresponding angles axiom.

- <u>Theorem 2</u>: If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

- <u>Theorem 3</u>: If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel. This is converse of Theorem 2 above.

Parallel Lines and a Transversal (Contd..)

- Theorem 4: If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

- Theorem 5: If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel to each other. This is converse of Theorem 5 above.

- Theorems 2, 3, 4 and 5 mentioned above can be easily proved using the Axiom Linear Pair of Angles are supplementary, Theorem Vertically Opposite Angles are equal and Axiom Corresponding Angles are equal.

Lines Parallel to the Same Line

- **Theorem 6**: Lines which are parallel to the same line are parallel to each other.

  Proof:

  In Fig. 1.14, line l || line m, line l || line n and line t is the transversal.

  Therefore, $\angle 1 = \angle 2$ and $\angle 1 = \angle 3$ (Using Corresponding Angles Axiom)
  So, $\angle 2 = \angle 3$ (as both are equal to $\angle 1$)
  We can say that, $\angle 2$ and $\angle 3$ are corresponding angles and they are equal.
  Therefore, you can say that
  line m || line m (Using Converse of Corresponding Angles Axiom)

  So, Lines m and n which parallel to line l have been proved to be parallel to each other.

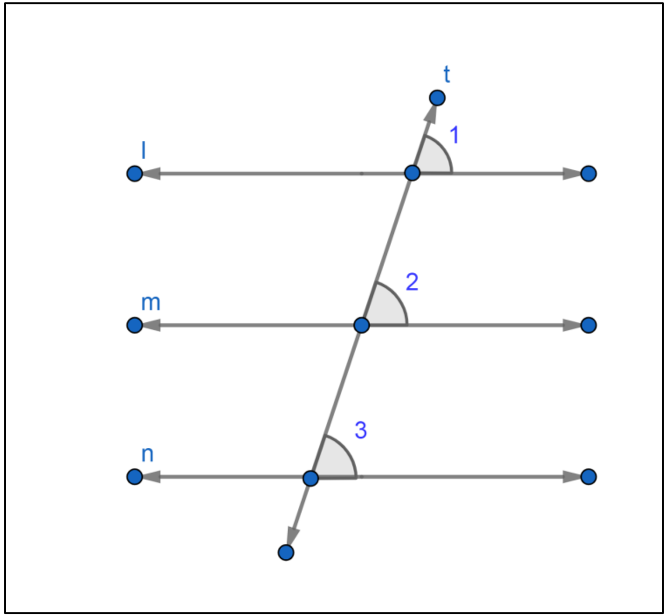


Fig. 1.14

Angle Sum Property of a Triangle

- **Theorem 7**: The sum of angles of a triangle is 180°.

Proof:

In Fig. 1.15, we are given a triangle ABC and DE is a line parallel to BC.

DE||BC and AB is the transversal, so $\angle 4=\angle 2$ (Eq. 1) (Pairs of alternate angles)
Similarly, DE||BC and AC is the transversal, so $\angle 5=\angle 3$ (Eq. 2)

Now, we can see that DAE is a line.
So, $\angle 4+\angle 1+\angle 5=180°$ (Eq. 3)
Substituting $\angle 4$ with $\angle 2$ and $\angle 5$ with $\angle 3$ in Eq. 3 using Eq. 1 and Eq.2, we get
$\angle 2+\angle 1+\angle 3=180°$

Thus, sum of angles of a triangle is 180°.

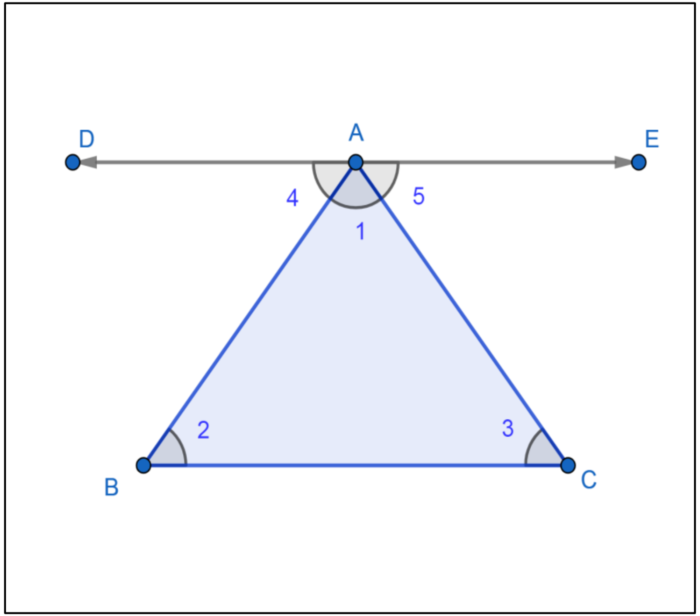


Fig. 1.15

Angle Sum Property of a Triangle (Contd..)

- **Theorem 8**: If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Proof:

In Fig. 1.16, we are given a triangle ABC and side AB is extended to AD.

Now using Angle Sum Property of Triangle,
$\angle 1 + \angle 2 + \angle 3 = 180^\circ$ (Eq. 1)
$\angle 3 + \angle 4 = 180^\circ$ (Eq. 2) (Using Linear Pair of Angles)
Now by Eq. 1 and 2, we can say that
$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$
Subtracting $\angle 3$ from both sides of the equation, we get
$\angle 1 + \angle 2 = \angle 4$

So, Theorem 8 is proved.

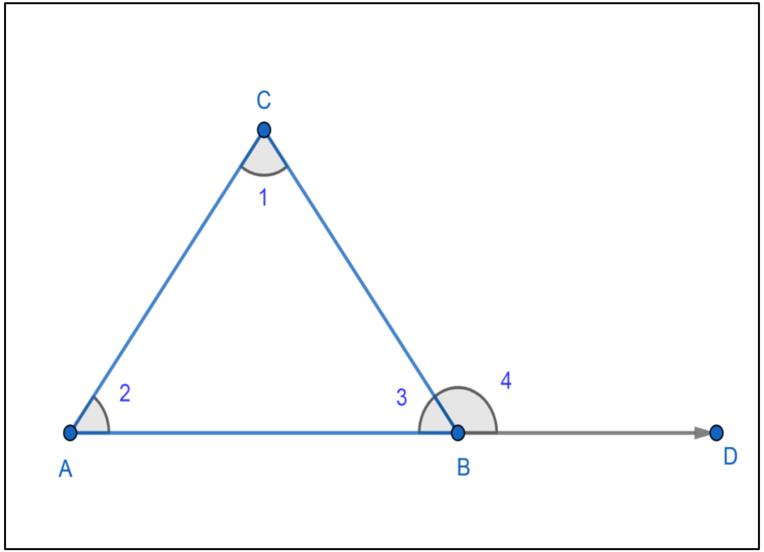


Fig. 1.16

Summary

- If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and vice versa. This property is called as the Linear pair axiom.
- If two lines intersect each other, then the vertically opposite angles are equal.
- If a transversal intersects two parallel lines, then

  (i) each pair of corresponding angles is equal,

  (ii) each pair of alternate interior angles is equal,

  (iii) each pair of interior angles on the same side of the transversal is supplementary.

Summary (Contd..)

- If a transversal intersects two lines such that, either
  (i) any one pair of corresponding angles is equal, or
  (ii) any one pair of alternate interior angles is equal, or
  (iii) any one pair of interior angles on the same side of the transversal is supplementary,
  then the lines are parallel.

- Lines which are parallel to a given line are parallel to each other.

- The sum of the three angles of a triangle is 180°.

- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

THANK YOU